Wave trapping and radiation by semi-immersed circular cylinders

J R Chaplin*
University of Southampton,
Southampton SO17 1BJ, UK
j.r.chaplin@soton.ac.uk

R Porter
University of Bristol,
Bristol BS8 1TW, UK

Highlights

For the first time laboratory measurements are presented of wave propagation in conditions in which Porter & Evans (2009) made the following predictions: (a) that waves incident on a floating semi-immersed cylinder would be totally reflected, and (b) that a pair of semi-immersed cylinders can exhibit a family of motion-trapped modes. The theory is extended to cover the situation in which the cylinders in the latter case are mechanically driven.

1. Introduction

Porter & Evans (2009) showed that incident waves of a particular frequency in deep water meeting a freely floating cylinder will be totally reflected. They used this remarkable result to construct examples of motion-trapped modes, in which a pair of identical cylinders, semi-immersed and floating freely would support a persistent localised time-harmonic oscillation such that no wave energy is radiated to infinity. The motion is assumed to be two-dimensional, and the cylinders are unconstrained and move in both heave and sway directions. Three parameters define the operation of this system: c/b being the ratio of cylinder radius c to half the separation of the cylinder centres, b; (ii) $kc \equiv \omega^2 c/g$ being the dimensionless wavenumber in terms of a radian frequency ω of the motion; (iii) X_1/X_2 being the ratio between the amplitudes of cylinder motion in sway and heave. The latter is, in general, a complex quantity which encodes both the relative amplitude and phase of the two components of motion. In the frequency domain, $X_1/X_2 = U_1/U_2$, the ratio of the complex velocities in sway and heave.

Table 1 of Porter & Evans (2009) summarises particular sets of values of these three parameters giving rise to these so-called motion-trapped modes (after McIver & McIver (2006)). They are reproduced in Table 1 below, with sketches to illustrate the motion in each case. Similar graphical depictions of the oscillation of both the cylinders and the surface were given in figures 5 and 6 of Porter & Evans (2009). However, the interpretation of how the cylinders move in that paper is wrong; the directions of motion of the cylinders were in each case measured in the wrong sense from the vertical.

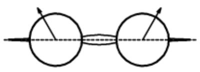
Type of mode	1st symmetric	1st anti-symmetric	2nd symmetric
kc	1.12567	1.12582	1.12590
c/b	0.60333	0.32808	0.22504
U_1/U_2	-0.57206	-0.56702	-0.56746
			

Table 1: Motion-trapped mode parameter sets, with data reproduced from Table 1 of Porter & Evans (2009). Sketches in the bottom row indicate the directions of the motion of the two cylinders when the one on the right is moving upwards. As c/b decreases more free surface oscillations exist between the cylinders.

To demonstrate these phenomena in the laboratory one would ideally displace the freely floating cylinders in some way and hope to observe a very slow decay in their motion. There are various practical problems with this however, and we chose instead to drive the cylinders mechanically (with the separations and orientations given in Table 1) with small amplitude harmonic displacements over a range of frequencies in each case. Disappearance of wave radiation would be a strong indication of at least wave trapping, if not of a motion-trapped mode. The distinction between these processes is discussed below.

2. Experimental arrangements

The experiments were carried out in a 17m long wave flume which is 420mm wide and operated with a still water depth of 700mm. At one end it is equipped with a flap-type wavemaker with active absorption, and at the other with a large volume of polyether foam. Wave reflections from either end are no more than a few percent of incident wave amplitudes, but in any case in most tests described here the measurements were completed before reflections arrived.

The two identical cylinders used on the experiments were sealed perspex tubes, ballasted internally with lead to make them float at half depth. They fitted across the width of the flume with a gap of about 1mm at each end. In those tests in which the cylinders were driven, they were mounted on descending arms connected to identical precision linear actuators comprising low-friction ballscrews driven by electrical servo motors in a closed digital control loop updated at 2.4kHz. The positional accuracy of the actuators is ±24 microns. Figure 1 shows the two cylinders mounted on the actuators in the flume.

Measurements of the motion of the water surface were made with conventional resistive parallel wire probes which were calibrated statically and dynamically in still water. In some cases the wave amplitudes to be measured were of the order of a millimetre, which is at the limit of the accuracy of these devices. For this reason we carried out additional calibrations in very small waves, in which signals from wire probes were compared with optical measurements of the motion of an adjacent spot on the water surface made by a vertical laser beam.

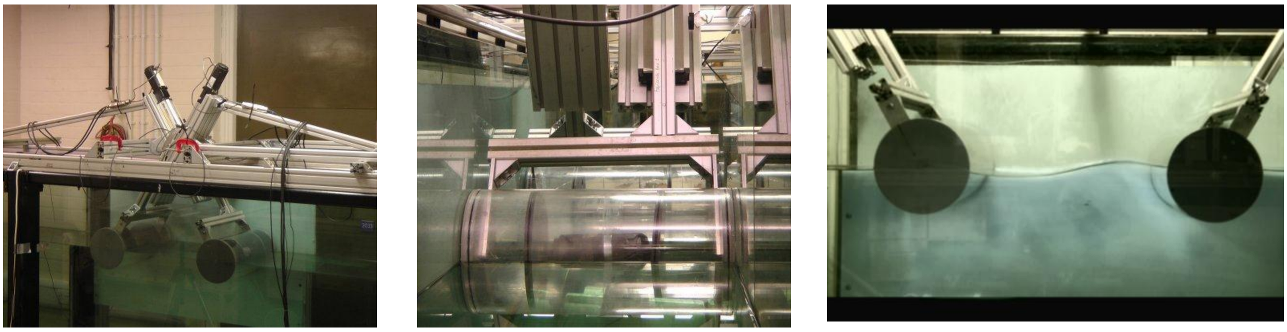


Figure 1. From left to right, the two cylinders mounted in the flume beneath the linear actuators; a view along the flume; the displaced water surface in the first anti-symmetric mode.

3. Scattering of incident waves by a freely floating cylinder

The linear predictions of Porter & Evans (2009) neglect the fact that in reality the cylinder drifts steadily away from the wavemaker owing to the second order force on it (against the small effect of mass transport in the flume). Consequently the frequency of waves that it encounters (and radiates) is slightly lower than that of the incident waves. And a wave probe on the wavemaker side of the cylinder records a signal that is modulated in amplitude both because of the arrival of waves of different frequencies, and because the origin of the reflected waves is receding. An example is shown in figure 2(a).

Predicted and measured reflection and transmission coefficients are shown as functions of kc in figure 2(b). According to the theory waves are totally reflected when $kc \approx 1.12593$. Measurements in figure 2(b) are in reasonable agreement with the theory but there is considerable loss of energy at higher frequencies. This is probably due to the cylinder's motion being damped by viscous resistance in the narrow gap between its ends and the flume walls.

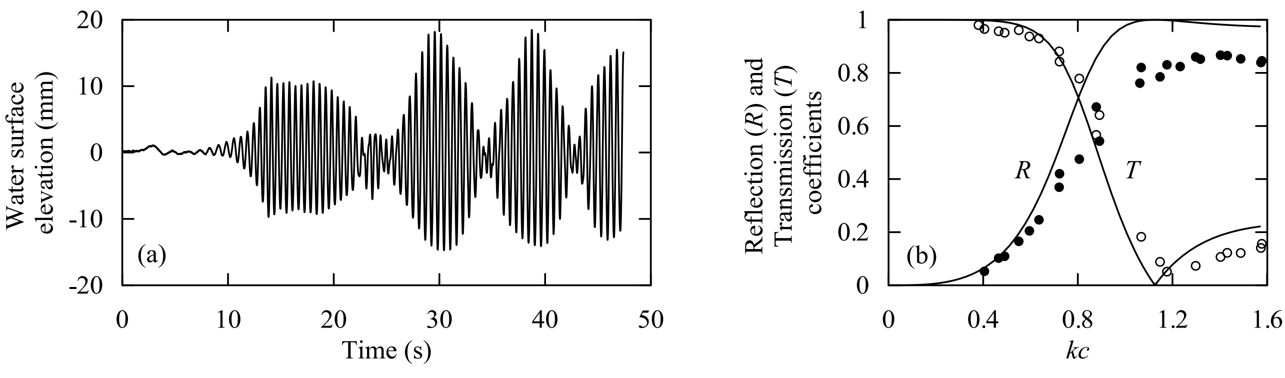


Figure 2. (a) A typical water surface elevation recorded by a stationary wave probe on the wavemaker side of the floating cylinder; kc = 1.233; (b) Reflection and transmission coefficients as functions of kc. Lines are from the theory of Porter & Evans (2009); points represent present measurements.

4. Forced oscillations of a pair of cylinders

Computations of radiated waves

The problem considered here is a variation of that addressed by Porter & Evans (2009). In the original paper the emphasis was on computing conditions on the spacing, frequency and motion paths of the cylinder under which motion-trapped modes occurred. Here, the cylinders are given fixed spacings and driven with fixed amplitudes along given motion paths. As the frequency is varied, the problem now is to determine, using the same semi-analytical techniques used by Porter & Evans (2009), the amplitudes of the waves radiated to infinity. In Porter & Evans (2009) the main ingredients used in their computation of motion-trapped modes were the components of the far-field radiated wave amplitudes for pairs of cylinders in either in-phase or anti-phase heave or surge motions (depending, respectively, on whether symmetric or anti-symmetric motion-trapped modes were being sought). The analytical details behind the calculations of these radiated wave amplitudes were given in a separate online technical report (Porter (2008)). For the problem here we also require the same components of heave and surge radiated wave amplitudes but, instead of using them in conditions for motion-trapped modes, we combine them by linear superposition with multipliers given by the heave and surge components of the driving motion. This results in the total far-field radiated wave amplitude for combined heave/surge motion which can easily be related to the free surface amplitudes. Of course, when the cylinder spacings, frequency and motion path coincide with the wave-free motion-trapped modes of Porter & Evans (2009) we find that there is no wave radiation to infinity.

Results are plotted in figure 3 in the form of the amplitudes a of radiated waves, normalised with respect to the amplitude of the cylinders' motion A. Plots (a), (b) and (c) (with magnified regions on the right) correspond to the conditions (cylinder separations and orientations of motion) listed in Table 1 for the first three motion-trapping modes.

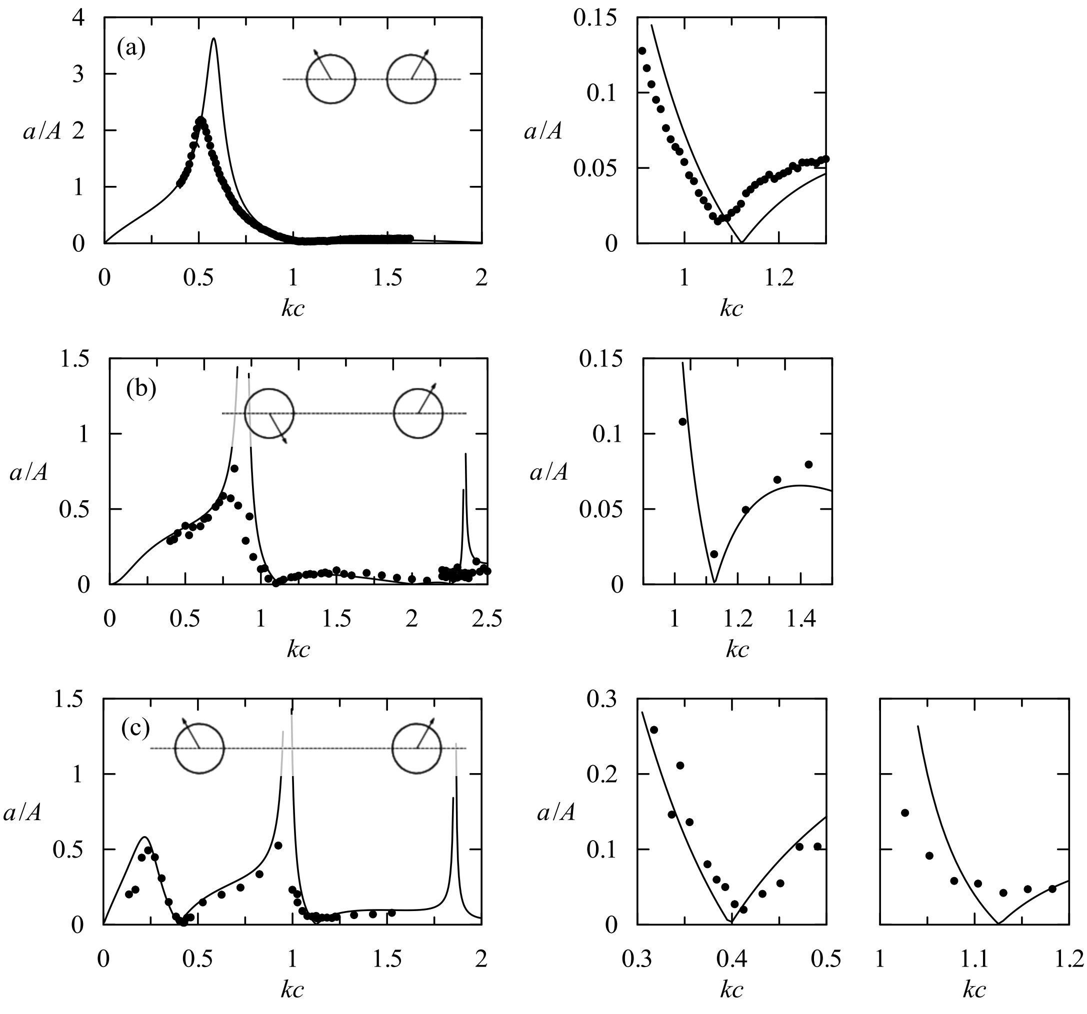


Figure 3. Predicted and measured amplitudes of radiated waves, normalised with respect to the amplitude of the cylinders' motion plotted as functions of frequency. Cases (a) (b) and (c) correspond respectively to the conditions identified in Table 1 with the first symmetric mode, the first anti-symmetric mode, and the second symmetric mode.

The lines touch the *kc* axis in each case at the point near *kc* = 1.125 identified in Table 1 as the conditions in which motion-trapping modes occur. Here the hydrodynamic forces on the cylinder exactly balance its acceleration and the force in the driving arms can be expected to be zero. Significantly, zero wave radiation occurs also at other frequencies which as far as is known do not correspond to motion-trapped modes. In these cases the inertia of the cylinder is not exactly balanced by the hydrodynamic forces on it and we expect to find that the force in the driving arms is not zero, but is in quadrature with their velocity.

It is not surprising that there are multiple frequencies at which wave radiation is zero. Porter & Evans (2011) showed that the far field wave amplitudes associated with heaving and surging only ever differ in phase by a multiple of π (a result independent of cylinder cross section, and applicable also to roll motions).

Measurements of radiated waves

Individual points in figure 3 represent measured radiated wave amplitudes. They are plotted against the dimensionless wavenumber *kc* calculated for the actual water depth in the flume, rather than $kc \equiv \omega^2 c/g$ as in the theory. In fact the difference is detectable only in the very few cases where $kc < 0.4$. Elsewhere it is reasonable to expect the effect of the finite water depth to be negligible. In the conditions of plot (a) the motion achieved a steady state very quickly and data collection could be completed before reflections arrived from the ends of the flume. When the cylinders were more widely spaced there was a much longer transient and useful data collection could begin only when wave motion in the whole flume had settled down. All the measurements follow the theory reasonably well but fail to exhibit the very large responses that occur in the theory over very narrow ranges of frequency. Also some wave motion could be detected several wavelengths away from the cylinders even at frequencies associated with wave trapping. Possible causes of this are slight errors in the inclinations of the driving arms (which could be positioned only to within about 1 degree of the required angles), jetting through the gaps at the ends of the cylinders, and the effects of the boundary layers on the cylinders.

Inadvertently we found another case of wave trapping, for which results are plotted in figure 4. Here the cylinder spacing was the same as for the first symmetric mode (as in figure 3(a)), but the angles of their trajectories about the vertical were reversed. Measurements of radiated waves fall close to the predicted zero near *kc* = 0.61.

On the whole the measurements provide useful validation of the theory to the extent that they agree (to within an reasonable degree of confidence) with the expected magnitudes of wave radiation. But this in itself does not entirely confirm that motion-trapped modes exist in the conditions identified in Table 1, and seen as the zeros near *kc* = 1.125 in figure 3. Equally the measurements cannot confirm that other zeros (like that in figure 4) are *not* associated with motion-trapped modes. Any such confirmation must await further experiments in which measurements are made of the forces in the actuator arms. We plan to present these results at the Workshop.

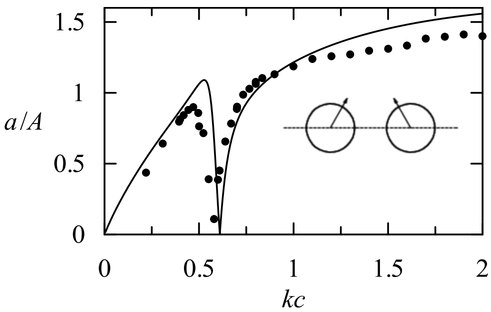


Figure 4. As for figure 3(a), again with the cylinders oscillating in phase, but with the angles of their trajectories from the vertical reversed.

References

McIver, P. & McIver, M. 2006, Trapped modes in the water-wave problem for a freely-floating structure. J. Fluid Mech., 558, 53–67.

Porter, R. 2008, The solution to water wave scattering and radiation problems involving semi-immersed circular cylinders. http://www.maths.bris.ac.uk/~marp/abstracts/semicyl techrep.html

Porter, R & Evans, D.V. 2009, Wave trapping by floating circular cylinders. J. Fluid Mech. 633 311–325

Porter, R & Evans, D.V. 2011, Estimation of wall effects on floating cylinders J. Eng Maths,70(1-3) 191–204.