The interaction of a submerged object with a Very Large Floating Platform

A.J. Hermans
Delft Institute of Applied Mathematics, TU Delft, Mekelweg 4, 2628 CD Delft, The Netherlands,
E-mail: a.j.hermans@ewi.tudelft.nl

1 Introduction

It was at the 11^{th} IWWWFB that the late professor Ohkusu [5] draw my attention to the problem of interaction of waves with very large flexible floating platforms. At that time I had the impression that an asymptotic approach, as in geometrical-optics (ray method), could be applicable. This turned out not to be a simple exercise. In [1] two numerical methods are shown to solve the two-dimensional floating platform for deep water, these approaches can be extended to finite water depth. At that point an asymptotic approach was still an open question mainly because the fact that water waves at finite depth can be written as a superposition of a progressing mode and infinitely many evanescent modes complicates the asymptotic significantly. However, it is possible to derive a semi-analytic method to solve the problem see [2, 3, 4]. Last year Sturove [6] looked after a more complicated problem. She studied the situation that not only a platform is present, but also a submerged cylinder. So the question is, whether our approach can be extended to this situation. It will be shown that this is possible indeed. The extension is simple if the submerged object is in front of the platform. If one realises why this is the case one also can try to extend the method to the case that the cylinder is underneath the platform. To find out how this can be done we first consider the effect of a point source below the free surface in front of and underneath the platform. For the cylinder underneath the platform we may use an extension as described in the multi-strip problem [3] or a combination with [1].

2 Mathematical formulation

We consider the two-dimensional interaction of an incident wave with a combination of a flexible floating dock or very large floating flexible platform (VLFP) with zero draft and a fixed submerged cylinder.

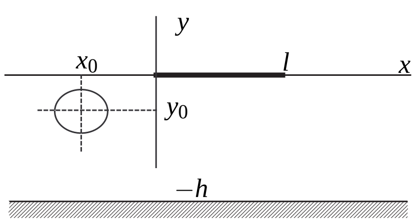


Figure 1: configuration

The fluid is ideal, so we introduce the velocity potential $\mathbf{V}(\mathbf{x},t) = \nabla\Phi(\mathbf{x},t)$, where $\mathbf{V}(\mathbf{x},t)$ is the fluid velocity vector. Hence $\Phi(\mathbf{x},t)$is a solution of the Laplace equation $\Delta\Phi = 0$ in the fluid, together with the linearized kinematic condition, $\Phi_y = \tilde{v}_t$, and dynamic condition, $p/\rho = -\Phi_t - g\tilde{v}$, at the mean water surface $y = 0$, where $\tilde{v}(x,t)$ denotes the free surface elevation, and ρ is the density of the water. The linearized free surface condition outside the platform, $y = 0$ and $x \in \mathcal{F}$, becomes:

$$\frac{\partial^2\Phi}{\partial t^2} + g\frac{\partial\Phi}{\partial y} = 0. \tag{1}$$

together with

$$\Phi_n = 0 \;\text{ at the cylinder } \mathcal{C} \quad \text{and } \Phi_y = 0 \;\text{ at } y = -h.$$

To describe the vertical deflection $\tilde{v}(x,t)$ of the platform, we apply the isotropic thin-plate theory and use the kinematic and dynamic condition to arrive at the following equation for Φ at $y = 0$ in the platform area $x \in \mathcal{P}$:

$$\left\{\frac{D}{\rho g}\frac{\partial^4}{\partial x^4} + \frac{m}{\rho g}\frac{\partial^2}{\partial t^2} + 1\right\}\frac{\partial\Phi}{\partial y} + \frac{1}{g}\frac{\partial^2\Phi}{\partial t^2} = 0. \tag{2}$$

We assume that the velocity potential is a time-harmonic wave function, $\Phi(\mathbf{x},t) = \phi(\mathbf{x})\, \mathrm{e}^{-\mathrm{i}\omega t}$. We introduce the following parameters: $K = \frac{\omega^2}{g}$, $\mu = \frac{m\omega^2}{\rho g}$, $\mathcal{D} = \frac{D}{\rho g}$. The potential of the undisturbed incident wave is given by:

$$\phi^{\text{inc}}(\mathbf{x}) = \frac{g\zeta_\infty}{\mathrm{i}\omega}\frac{\cosh(k_0(y+h))}{\cosh(k_0 h)}\exp(\mathrm{i}k_0 x) \tag{3}$$

where ζ_∞ is the wave height in the original coordinate system, ω the frequency, while the wave number k_0 is the positive real solution of the dispersion relation, $k_0\tanh(k_0 h) = K$, for finite water depth. If the Green's function obeys the free surface boundary condition, the expression for the total potential becomes, :

$$
\begin{aligned}
2\pi\phi(x,y) = 2\pi\phi^{\text{inc}}(x,y) - \int_C \left\{ \phi(\xi,\eta)\frac{\partial\mathcal{G}(x,z;\xi,\eta)}{\partial n} - \frac{\partial\phi(\xi,\eta)}{\partial n}\mathcal{G}(x,z;\xi,\eta)\right\}\,\mathrm{d}s \\
+ \int_0^l \left(\phi(\xi,0)\frac{\partial\mathcal{G}(x,y;\xi,0)}{\partial\eta} - \frac{\partial\phi(\xi,0)}{\partial\eta}\mathcal{G}(x,z;\xi,0)\right)\,\mathrm{d}\xi.
\end{aligned}
\tag{4}
$$

In the case of diffraction the second term in the first integral cancels. If the object oscillates in still water this term is given by the motion, for instance heave or sway.
A crucial step is the choice of the Green's function. It is possible to derive the Green's function $\mathcal{G}(x,y;\xi,\eta)$ by means of a Fourier transform with respect to the x-coordinate. It has the form:

$$
\mathcal{G}(x,y;\xi,\eta) = \int_{\mathcal{L}'} \frac{1}{\gamma}\frac{K\sinh\gamma y + \gamma\cosh\gamma y}{K\cosh\gamma h - \gamma\sinh\gamma h}\cosh\gamma(\eta+h)\ \mathrm{e}^{\mathrm{i}\gamma(x-\xi)}\,\mathrm{d}\gamma \quad \text{for } y>\eta
\tag{5}
$$

If we close the contour of integration in the complex $\gamma-$plane we obtain the complex version of formula (13.34), as can be found in Wehausen and Laitone [7]

$$
\mathcal{G}(x,y;\xi,\eta) = -2\pi\mathrm{i}\sum_{i=0}^{\infty}\frac{1}{k_i}\frac{k_i^2-K^2}{hk_i^2-hK^2+K}\cosh k_i(y+h)\cosh k_i(\eta+h)\ \mathrm{e}^{\mathrm{i}k_i|x-\xi|},
\tag{6}
$$

where k_i are the zeros, in the complex γ plane, of the dispersion relation $K\cosh\gamma h - \gamma\sinh\gamma h = 0$. k_0 is the positive real value representing the progressing wave contribution. $k_i,\ i=1,2,\cdots$ are the positive imaginary zeros, representing the evanescent modes. We shall use (5) for $\mathbf{x}$ at the platform for the integral along the platform and it residue expansion for the integral along the cylinder $\mathcal{C}$. For the distribution along the cylinder we use a different expression for the same Green's function, see the same reference [7]:

$$
\mathcal{G}(x,y;\xi,\eta) = \log(\frac{r}{h}) + \log(\frac{r_2}{h}) + \int_{\mathcal{L}'}\left\{\frac{\gamma+K}{\gamma}\frac{\mathbf{e}^{-\gamma h}\cosh\gamma(y+h)\cosh\gamma(\eta+h)\cos\gamma(x-\xi)}{K\cosh\gamma h-\gamma\sinh\gamma h} + \frac{\mathbf{e}^{-\gamma h}}{\gamma}\right\}\mathrm{d}\gamma,
\tag{7}
$$

where r_2 is the distance to the mirror of the source in the fluid domain with respect to the bottom. The contribution of the cylinder consists of a source distribution. Therefore we study the effect on the platform of an isolated source with unit strength first.

3 Unit source

When we consider a source (strength=$\frac{g}{\mathrm{i}\omega}$) in the fluid domain in front of the platform expression (6) represents the field underneath the progressing and evanescing to the right. If the source is placed underneath the platform one must distinguish distributions progressing and evanescing to left and right. The equation for the deflection of the platform becomes

$$
\begin{aligned}
2\pi\left(\mathcal{D}\frac{\partial^4}{\partial x^4} - \mu + 1\right)w(x) + K\int_0^l \mathcal{G}(x,0;\xi,0)\left\{\mu - \mathcal{D}\frac{\partial^4}{\partial\xi^4}\right\}w(\xi)\,\mathrm{d}\xi = \\
= -2\pi\mathrm{i}\sum_{i=0}^{\infty}\frac{1}{k_i}\frac{k_i^2-K^2}{hk_i^2-hK^2+K}\cosh k_i h\cosh k_i(y_0+h)\ \mathrm{e}^{\mathrm{i}k_i(x-x_0)}.
\end{aligned}
\tag{8}
$$

If an incident wave is present we have to add $2\pi\mathbf{e}^{\mathrm{i}k_0x}$ to the right hand side.
We introduce the following expansion for $w(x)$:

$$
w(x) = \sum_{n=0}^{N+1}\left(a_n\mathbf{e}^{\mathrm{i}\kappa_n x} + b_n\mathbf{e}^{-\mathrm{i}\kappa_n(x-l)}\right)
\tag{9}
$$

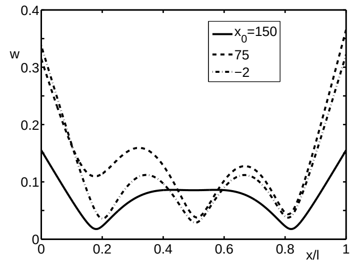
In expression (6) we use (3) as Green's function, doing so we can integrate with respect to ξ first. Closing the contour in the complex γ-plane leads to the dispersion relation for the plate:

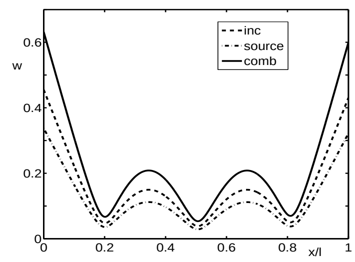
$$
(\mathcal{D}\kappa^4 - \mu + 1)\tanh\kappa h = K
\tag{10}
$$

together with a $2N$ equations of the $2N+4$ unknowns a_n and b_n:

$$
\begin{aligned}
\sum_{n=0}^{N+1}(\mathcal{D}\kappa_n^4-\mu)\left[\frac{a_n}{\kappa_n-k_i} - \frac{b_n\mathbf{e}^{\mathrm{i}\kappa_n l}}{\kappa_n+k_i}\right] &= \frac{k_i^2-K^2}{k_i^2K}\cosh k_i h\cosh k_i(y_0+h)\mathbf{e}^{-\mathrm{i}k_ix_0} \\
\sum_{n=0}^{N+1}(\mathcal{D}\kappa_n^4-\mu)\left[\frac{-a_n\mathbf{e}^{\mathrm{i}\kappa_n l}}{\kappa_n+k_i} + \frac{b_n}{\kappa_n-k_i}\right] &= 0, \qquad .
\end{aligned}
\tag{11}
$$

for $i = 0, \cdots, N-1$. The conditions at the endpoints of the platform $\frac{\mathrm{d}^2 w}{\mathrm{d}x^2} = \frac{\mathrm{d}^3 w}{\mathrm{d}x^3} = 0$ result in four equations. Hence for $x_0 < 0$ the problem has been solved If the source is at $x_0 > 0$ the source potential has a different of behaviour depending of the sign of $x - x_0$. This problem can be solved by cutting the platform at $x = x_0$ and treat each platform in a similar way as in [3]. At the artificial discontinuity one requites continuity of The deflection and its first three derivatives.





In the figure on the left hand side the deflection, of the platform, induced by a point source at the location indicated is shown. The length of the platform is 300 m., $\mathcal{D} = 10^7$ m.[4] and $\frac{\omega^2}{g} = 2\pi/60$, hence the *deep water* wavelength is 60 m. The figure on the right hand side shows the deflection for an incident wave, a source at $x_0 = -2$, and a combination of incident wave and source. In both figures $y_0 = -\,1.5$.

4 Interaction with circular object

In this section we solve equation (4), with a convenient choice of the source function. We consider the situation that the submerged cylinder, $\mathcal{C}$, is in front of or behind the platform, $\mathcal{P}$, completely. The structure of the matrix becomes:

$$\left(\begin{array}{c|c} \mathcal{C} \rightarrow \mathcal{C} & \mathcal{P} \rightarrow \mathcal{C} \\ np \text{ x } np & (2N+4) \text{ x } np \\ \hline \mathcal{C} \rightarrow \mathcal{P} & \mathcal{P} \rightarrow \mathcal{P} \\ np \text{ x } (2N+4) & (2N+4) \text{ x } (2N+4) \end{array}\right).$$

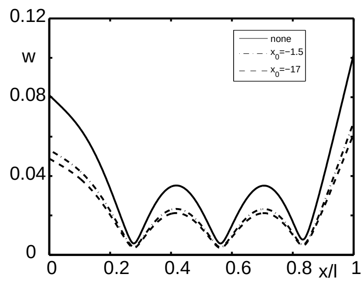
We have divided the circle $\mathcal{C}$ in np segments, so we have np unknown values of the source strengths. Here we use the third expression for the Green's function.
At the platform $\mathcal{P}$ we have the matrix as before for the $2N+4$ unknown coefficient of the series. For $n = 0, \cdots, N+1$ the effect of $\mathcal{P}$ on $\mathcal{C}$ becomes

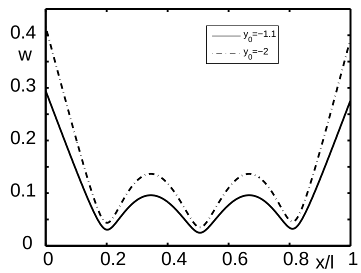
$$-\frac{2\pi K}{\mathcal{D}} \sum_{i=0}^{N-1} (\mathcal{D}\kappa_n^4 - \mu) k_i \frac{\cosh k_i(y+h)}{(k_i^2 h + K - K^2 h)\cosh k_i h}\, \mathrm{e}^{-\mathrm{i}k_i x} \left[a_n \frac{\mathrm{e}^{\mathrm{i}(\kappa_n + k_i)l} - 1}{\kappa_n + k_i} + b_n \frac{\mathrm{e}^{\mathrm{i}\kappa_n l} - \mathrm{e}^{\mathrm{i}k_i l}}{\kappa_n - k_i} \right].$$

The influence of unknown source strength on $\mathcal{C}$ on the platform $\mathcal{P}$ for $i = 0, \cdots, N-1$ follows from

$$\int_{\mathcal{C}} \phi(\xi,\eta) \frac{\partial \mathcal{G}_i(x,z;\xi,\eta)}{\partial n}\, \mathrm{d}s \quad \text{with} \quad \mathcal{G}_i(x,y;\xi,\eta) = -\frac{1}{k_i} \frac{k_i^2 - K^2}{hk_i^2 - hK^2 + K} \cosh k_i(y+h) \cosh k_i(\eta+h)\, \mathrm{e}^{\mathrm{i}k_i|x-\xi|}.$$

The righthand side is obtained accordingly.
If the cylinder is underneath the platform we suggest an approach similar to the one as described in [1]. We consider a circular cylinder with radius one and centre at (x_0, y_0). For the fixed cylinder we first compute the deflection of the platform induced by an incident wave with unit wave height. Next we show the influence of the moving platform on the value of the added mass and damping, as defined by Sturova [6] of the object in front of the platform. We compare two





The deflection is shown in the figure on the left for $\frac{\omega^2}{g} = 2\pi/30$ and two values of x_0 and fixed $y_0 = -1.5$, while the one on the right shows results for $\frac{\omega^2}{g} = 2\pi/60$ and different values of y_0 and fixed $x_0 = -1.5$.

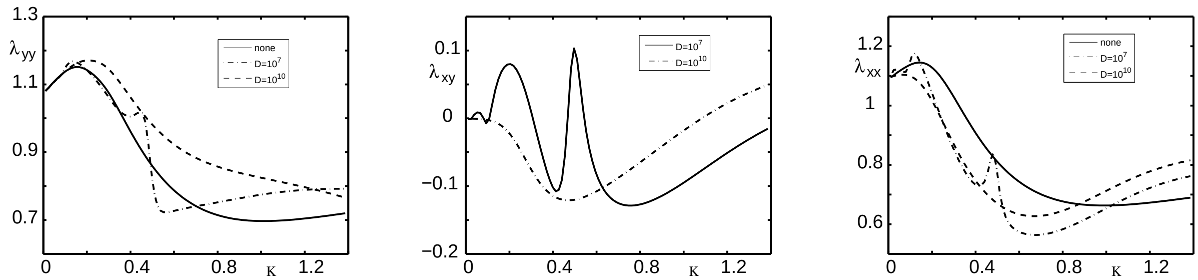


Figure 2: Added mass coefficients

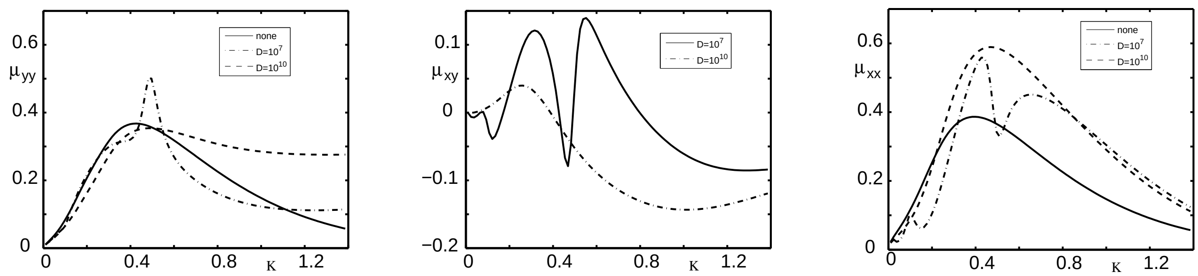


Figure 3: Damping coefficients

values of the added mass in heave, surge and the combined one for two values of $\mathcal{D}$, 10^7 m.^{4} and 10^{10} m.^{4} respectively, for an circular object at $(x_0, y_0) = (-1.5, -2)$.
The oscillations in the results at the lower value of the flexural rigidity, $\mathcal{D} = 10^7$ m. are due to the oscillatory motion of the flexible platform. For $\mathcal{D} = 10^{10}$ m. the results may be compared with the results of Sturova [6], but one must realize that the platform is still moving.
We see that the motion of the platform plays in the plays an important role in the coefficients

A suggestions for extension of the method for $\mathcal{C}$ underneath $\mathcal{P}$:

- One may make use of the fact that the dimension (in the $x-$direction) of the cylindrical object is small compared to the length of the platform. The same splitting procedure as described in the case of the point source may be used. Some results will be shown.

- For a large object or small platform one may solve the plate problem in a way as described in reference (1). I suggest to use the expansion in orthogonal modes, because then the integration with respect to x along the platform may be carried out analytically. The resulting integrals can be computed numerically as described in the reference.

References

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