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Multi-resonant compressible wave energy devices

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Highlights

- We propose two multi-resonant wave energy devices with compressible volumes. One is fixed to the seabed. The other is floating.
- Linear mathematical models, assuming no losses, predict that devices with displaced volumes of about 3000 m^3 are capable of absorbing 80% of the theoretical limit for a wave period range of 5 seconds.

1 Introduction

A heaving body with a compressible displaced volume can have a longer resonant period than if it were rigid. Motivated by this idea, Farley [2] proposed a compressible wave energy device in the form of a heaving wedge which opens and closes as it heaves. Compared to their rigid counterparts, such devices have a lower restoring stiffness, which offers them two advantages. First, they do not have to be large to resonate with the prevailing waves, which means a cost reduction. Second, a low stiffness is associated with a broad resonance bandwidth.

Here we propose a new idea of coupling the deformation of the compressible volume with a water column, which is not only necessary to provide a restoring force to the compressible volume, but also serves to broaden the response of the device by making it a coupled resonant system [1]. Two new compressible devices are proposed: one is fixed to the seabed and thus is more suited for nearshore locations, while the other is floating and therefore can operate further offshore (see Fig. 1). The idea is to keep the design relatively simple, to reduce cost.

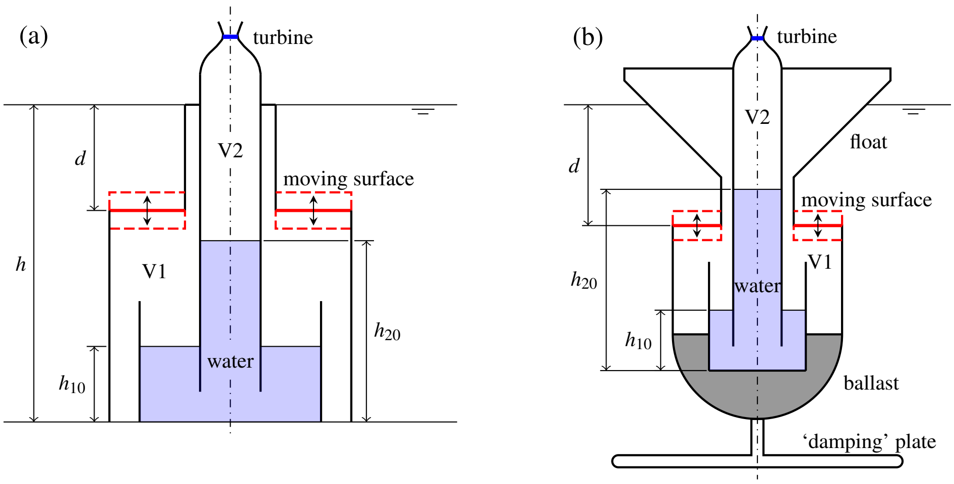

Figure 1: Sketch of the (a) bottom-fixed and (b) floating devices. The moving surface is free to move up and down. The sketch is representative only and not intended as a practical design.

2 Bottom-fixed device

The main components of the bottom-fixed device are a submerged compressible volume V1 with a moving surface on top, a water column, and an air turbine for power take-off. The moving surface is assumed to be a rigid horizontal surface, free to move up and down. Under wave excitations, the moving surface and the water column, which are coupled via the compressible volume V1, oscillate, creating an air flow which drives the turbine. Here we consider an axisymmetric device, although it may in general be of any shape. In practice, the moving surface may be constructed from lightweight materials, and it may possibly be connected to the walls of V1 in the manner of a loudspeaker diaphragm. Also, the bends in the water column and any sharp edges need to be streamlined to reduce losses. It is not necessary for the entire V1 and the water column to be submerged; part of V1 and the water column may be located onshore or above the water.

At equilibrium, the volume and pressure in V1 are equal to V_{10} and $p_0 = \rho g d + p_{\mathrm{atm}}$, where d is the submergence of the moving surface. The equilibrium volume and pressure in V2 are equal to V_{20} and p_{atm}, while the equilibrium water column levels are related as $h_{20} = h_{10} + d$. We consider time-harmonic motions of small amplitude, with the complex factor $\mathrm{e}^{\mathrm{i}\omega t}$ applied to all oscillatory quantities. Assuming an isentropic air pressure-density relation $p(V/m)^{\gamma} = \text{constant}$, with $\gamma = 1.4$, for volume V1 we have

$$p_1 = -\gamma p_0 V_1/V_{10}, \tag{1}$$

while for volume V2,

$$V_2/V_{20} = m_2/m_{20} - p_2/(\gamma p_{\mathrm{atm}}). \tag{2}$$

The flow through the turbine is idealised with the following linear relationship:

$$-\mathrm{i}\omega m_2 = F p_2, \tag{3}$$

where F is the mass flow through the turbine for a unit pressure difference. Assuming that the water is incompressible, the amplitudes of the outer and inner water column levels, h_1 and h_2, are related by the cross-sectional areas S_{t1} and S_{t2}:

$$S_{t1} h_1 = -S_{t2} h_2. \tag{4}$$

The volume amplitudes in V1 and V2 are given as

$$V_1 = -h_1 S_{t1} + \xi_7 S_1 = h_2 S_{t2} + \xi_7 S_1 \tag{5}$$

$$V_2 = -h_2 S_{t2} = h_1 S_{t1}. \tag{6}$$

where ξ_7 and S_1 are respectively the vertical displacement and the area of the moving surface, and we have used (4) to obtain the second equalities. Finally, the equation of motion for the water column, assuming no losses, is given as

$$p_1 - p_2 = -\omega^2 \rho\, h_2 \left[d + h_{10} \left(1 + \frac{S_{t2}}{S_{t1}} \right) \right] + \rho g\, h_2 \left(1 + \frac{S_{t2}}{S_{t1}} \right). \tag{7}$$

Using (1) to (6), we may eliminate p_1 and p_2 in (7) to obtain an equation of motion for the water column in terms of the variables h_2 and ξ_7. We can then write the coupled equations of motion for the moving surface and the water column in matrix form:

$$\left\{ -\omega^2 \begin{bmatrix} m_{77} & 0 \\ 0 & \rho\,(d + h_{10} f) \end{bmatrix} + \mathrm{i}\omega \begin{bmatrix} R_{77} & 0 \\ 0 & \dfrac{\rho g}{\omega} \operatorname{Im}\{L\} \end{bmatrix} + \begin{bmatrix} -\rho g S_1 + S_1^2 \dfrac{\gamma p_0}{V_{10}} & S_1 S_{t2} \dfrac{\gamma p_0}{V_{10}} \\ S_1 \dfrac{\gamma p_0}{V_{10}} & \rho g f + S_{t2} \dfrac{\gamma p_0}{V_{10}} + \rho g \operatorname{Re}\{L\} \end{bmatrix} \right\} \begin{bmatrix} \xi_7 \\ h_2 \end{bmatrix} = \begin{bmatrix} F_{e7} \\ 0 \end{bmatrix}, \tag{8}$$

where m_{77} and R_{77} are the added mass and radiation damping coefficients of the moving surface, and F_{e7} is the wave exciting force on the moving surface. The non-dimensional quantities f and L are defined as follows:

$$f = 1 + \frac{S_{t2}}{S_{t1}} \tag{9}$$

$$L = \frac{S_{t2}}{\rho g V_{20} \left(\dfrac{F}{\mathrm{i}\omega m_{20}} + \dfrac{1}{\gamma p_{\mathrm{atm}}} \right)}. \tag{10}$$

Here we have assumed that, apart from its added mass, the moving surface has no mass of its own. The restoring stiffness of the moving surface is the sum of the negative hydrostatic stiffness and the positive pneumatic stiffness of V1.

Equation (8) can be solved for ξ_7 and h_2. Upon finding p_2 using (2), (3), and (6), we can obtain the mean absorbed power in regular waves as

$$P = \frac{F}{2\rho_{\mathrm{air}}} |p_2|^2. \tag{11}$$

3 Floating device

The main components of the floating device are the same as those of the bottom-fixed device, except that now, in addition, we have a float at the top, a 'damping' plate at the bottom, and an amount of ballast to balance the buoyancy force. Again, it is not necessary for V1 to be entirely submerged; part of it may be located above water. We consider an axisymmetric device and consider pure heave motion. Compared to the bottom-fixed device, we now have an additional coupling from the heave of the float. If the displacement of the moving surface and the levels of the water column are defined relative to the float, then equations (1) to (6) apply without change. Equation (7) is, however, modified to include a coupling from the acceleration of the float:

$$p_1 - p_2 = -\omega^2 \rho\, h_2 \left[d + h_{10} \left(1 + \frac{S_{t2}}{S_{t1}} \right) \right] + \rho g\, h_2 \left(1 + \frac{S_{t2}}{S_{t1}} \right) - \omega^2 \rho d\, \xi_3, \tag{12}$$

where ξ_3 is the heave of the float. In addition, due to the acceleration of the water column, the float experiences a vertical force which is given as $\omega^2 h_2 \rho d S_{t2}$.

We may then write the coupled equations of motion for the float, the moving surface, and the water column as follows:

$$\left\{ -\omega^2 \begin{bmatrix} M+m_{33} & m_{37} & \rho d S_{t2} \\ m_{73} & m_{77} & 0 \\ \rho d & 0 & \rho\left(d+h_{10}f\right) \end{bmatrix} + \mathrm{i}\omega \begin{bmatrix} R_{33} & R_{37} & 0 \\ R_{73} & R_{77} & 0 \\ 0 & 0 & \dfrac{\rho g}{\omega}\,\mathrm{Im}\{L\} \end{bmatrix} + \begin{bmatrix} \rho g S_2 & -\rho g S_1 & 0 \\ -\rho g S_1 & -\rho g S_1 + S_1^2 \dfrac{\gamma p_0}{V_{10}} & S_1 S_{t2} \dfrac{\gamma p_0}{V_{10}} \\ 0 & S_1 \dfrac{\gamma p_0}{V_{10}} & \rho g f + S_{t2} \dfrac{\gamma p_0}{V_{10}} + \rho g\,\mathrm{Re}\{L\} \end{bmatrix} \right\} \begin{bmatrix} \xi_3 \\ \xi_7 \\ h_2 \end{bmatrix} = \begin{bmatrix} F_{e3} \\ F_{e7} \\ 0 \end{bmatrix}, \tag{13}$$

where f and L are as defined in (9) and (10). Here, M is the mass of the device, S_2 is the water plane area, m_{33} and R_{33} are the heave added mass and radiation damping coefficients of the device if it were rigid, m_{77} and R_{77} are the added mass and radiation damping coefficients of the moving surface if the device were fixed, $m_{37} = m_{73}$ and $R_{37} = R_{73}$ are the coupled added mass and radiation damping coefficients, and F_{e3} is the heave exciting force on the device if it were rigid. As in the bottom-fixed device, the mean absorbed power can be obtained using (11).

4 Results and discussions

We present numerical results for the devices with dimensions as shown in Fig. 2. The dimensions have not been numerically optimised. The added mass, radiation damping, and wave exciting force coefficients are computed using WAMIT [4], where the generalised body modes [3] option is used to define the relative displacement of the moving surface.

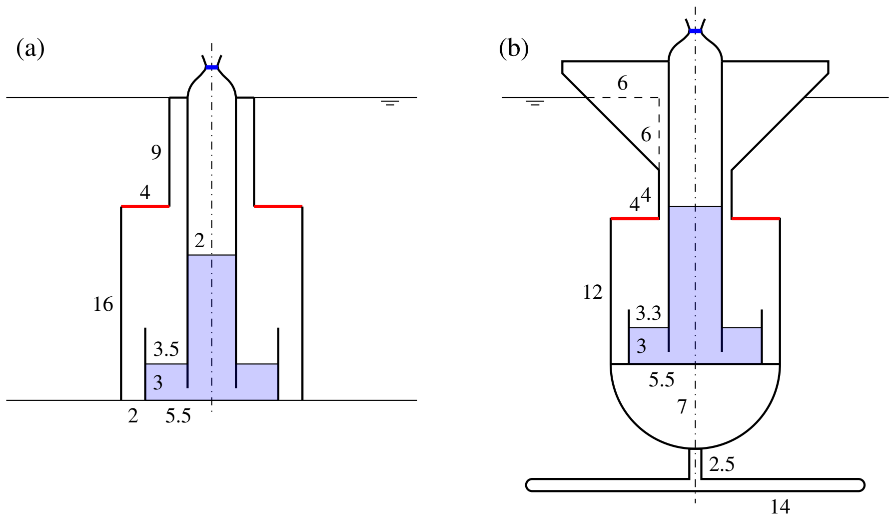

Figure 2: Dimensions (in m) of the (a) bottom-fixed and (b) floating devices. The total displaced volume of the bottom-fixed device is 3174 m^3, and $V_{10} = 2612$ m^3. The total displaced volume of the floating device, neglecting the 'damping' plate, is 3414 m^3, and $V_{10} = 3086$ m^3. The water depth is assumed to be infinite for the floating device case.

There is a difference in the behaviour of the two devices at the zero-frequency limit. The normalised heave displacement of the float, $|\xi_3/A|$, goes to one, while the normalised displacement of the moving surface relative to the float, $|\xi_7/A|$, goes to zero (Fig. 4a). This is because for long waves, the floating device moves together with the wave. For the fixed device, however, the displacement of the moving surface at the zero-frequency limit is finite (Fig. 3a) because it moves against a fixed reference.

With constant F, two response peaks are evident for the bottom-fixed device, as would be expected from a two-degree-of-freedom system. For the floating device, we expect to see three peaks as we now have a three-degree-of-freedom system. However, only two peaks are visible in Fig. 4, the third peak being at a frequency lower than the considered range. The trough between the two visible peaks for both the fixed and floating devices corresponds to the water column resonance when the moving surface is fixed relative to rest of the structure. This resonant frequency may be derived as

$$\omega_0 = \sqrt{g/l}, \tag{14}$$

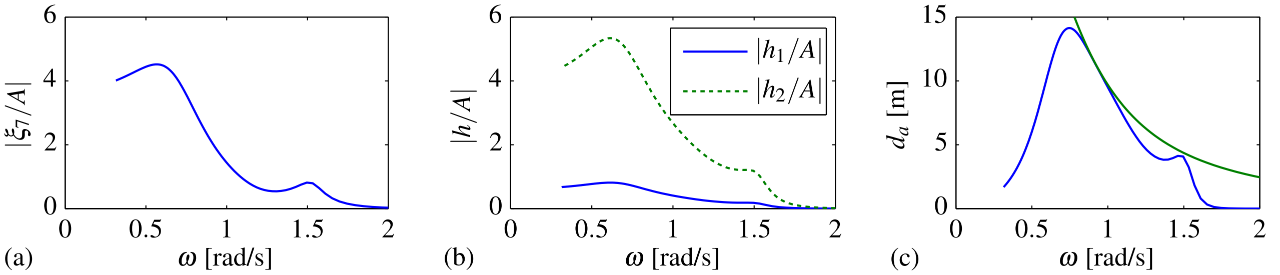

Figure 3: Numerical results for the bottom-fixed device as shown in Fig. 2: (a) displacement, per unit incident wave amplitude, of the moving surface, for $F = 0.0015$ ms; (b) corresponding displacements, per unit incident wave amplitude, of the water column levels; (c) corresponding absorption width, compared to the theoretical limit $\lambda/2\pi$ (descending line).

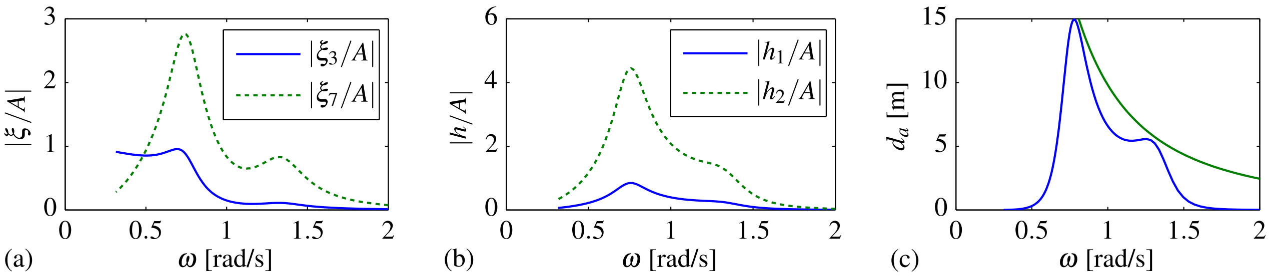

Figure 4: Numerical results for the floating device as shown in Fig. 2: (a) displacements, per unit incident wave amplitude, of the float and of the moving surface relative to the float, for $F = 0.003$ ms; (b) corresponding displacements, per unit incident wave amplitude, of the water column levels relative to the float; (c) corresponding absorption width, compared to the theoretical limit $\lambda/2\pi$ (descending line).

where

$$l = \begin{cases} \dfrac{d + h_{10} f}{f + S_{t2}/S_{p1} + S_{t2}/S_{p2}} & \text{when } F = 0 \text{ (V2 is closed)} \\ \dfrac{d + h_{10} f}{f + S_{t2}/S_{p1}} & \text{when V2 is completely open to the atmosphere,} \end{cases} \tag{15}$$

with $S_{p1} = V_{10}/[\gamma(d + p_{\mathrm{atm}}/\rho g)]$ and $S_{p2} = V_{20}/(\gamma p_{\mathrm{atm}}/\rho g)$. For the bottom-fixed device, then, ω_0 ranges from 1.00 rad/s when V2 is open, to 1.36 rad/s when V2 is closed. For the floating device, ω_0 ranges from 0.98 rad/s to 1.35 rad/s. From Figs. 3 and 4 we see that, for each device, the frequency of the trough lies within the above corresponding range.

The absorption widths for both devices are close to the theoretical limit $\lambda/2\pi$ for a wide range of wave frequencies, even with constant F. The absorption width of the floating device is, however, narrower than the fixed device. This is because for the floating device, there is no fixed reference against which the moving surface and the water column move. For low frequencies, i.e. for $\omega < \omega_0$, the water column displacement h_2 moves in phase with ξ_3. The decay of the response curve to zero at the low-frequency end is governed by the resonant frequency of the device if it were rigid. The lower the resonant frequency is, the lower is the frequency at which the response curve decays to zero. The 'damping' plate serves to increase the added mass of the rigid device, thereby shifting its resonant frequency to a much lower value than ω_0. In reality, there would be considerable damping arising from the 'damping' plate, which would reduce the absorbed power. However, we think that since power absorption is mainly through the moving surface, such reduction would be smaller than it would be if power were to be absorbed through the heave of the whole device.

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