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The Application of Velocity Decomposition to Fully-Submerged Free-Surface Problems

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Highlights

- The total velocity vector which satisfies the Navier-Stokes problem for lifting and free-surface flow is decomposed into irrotational and vortical components. The irrotational component is modeled with a velocity potential.
- The *viscous* potential is the solution to a modified velocity potential boundary-value problem and satisfies the viscous fluid equations outside of the rotational regions of the flow.
- The velocity decomposition approach is applied to solve for the free-surface flow over a fully-submerged foil offering accuracy and computational efficiency compared to a conventional Navier-Stokes numerical solution.

Introduction

The work presented in this paper is the continuation of work previously presented at the IWWWFB by Edmund et al. (2011) and Rosemurgy et al. (2013). The goal of this research is to use the principle of velocity decomposition to develop a method which provides the solution to the Navier-Stokes problem in the entire fluid domain, while only solving the Navier-Stokes equations in the region of the flow that is rotational. In our decomposition, the velocity vector is expressed using a Helmholtz-like decomposition as the sum of an irrotational component and a vortical component. In most external flows encountered in naval hydrodynamics, the majority of the flow is irrotational and can be represented using a velocity potential. The rotational flow is confined to the body-boundary layer and the wake downstream of the body. If a velocity potential is found which satisfies the Navier-Stokes problem *directly* outside of the vortical regions of the flow, then the region over which the Navier-Stokes equations must be numerically solved can be greatly reduced.

It is well known that the velocity potential which satisfies the non-penetration condition on the body (the *inviscid* potential) does not satisfy the Navier-Stokes problem, even outside of the rotational regions of the flow. Following Morino (1986), Edmund (2012) developed and numerically implemented the body-boundary condition for the *viscous* potential as used in this work. The body-boundary condition for the viscous potential includes the effects of viscosity on the body and in the wake. This boundary condition is based on the principle of conservation of mass and is general enough to be applied to steady, 2D and 3D, and fully separated flows.

In the current research, the viscous potential is further developed for lifting and free-surface flows. In free-surface problems where the body is fully-submerged, the viscous fluid equations are solved on a fluid domain which is entirely submerged and all free-surface effects are introduced through the velocity potential. In Rosemurgy et al. (2012), the velocity decomposition approach was applied to the flow over a bottom-mounted bump and the exact non-linear free-surface condition was satisfied. The fully non-linear free-surface condition has not yet been implemented for the lifting problem; the linear free-surface condition will be satisfied in this work.

The viscous potential is further modified to solve the lifting problem. One of the defining characteristics of a viscous lifting flow is the possible presence of separation on the suction side of a foil which results in a loss of lift. Another characteristic of lifting flows is the asymmetry which exists in the rotational wake. In Rosemurgy et al. (2013), a velocity decomposition approach for deeply-submerged lifting flow was presented. Specifically, an approach to account for the asymmetry in the wake was developed. This approach was successful in capturing the asymmetry of the flow in the wake region but did not account for the loss of lift due to separation. In the current work, an improved approach to capture the wake asymmetry is presented and the viscous potential is also modified to account for the loss of lift due to separation.

Problem Formulation

The Navier-Stokes problem is defined as the Navier-Stokes equations (1) and (2) with appropriate boundary conditions. In this work, the 2D steady, incompressible Navier-Stokes equations are solved.

$$\nabla \cdot \mathbf{u} = 0 \tag{1}$$

$$\nabla \cdot \mathbf{u} \otimes \mathbf{u} = -\nabla p/\rho + \nabla \cdot \nu \ \left(\nabla \otimes \mathbf{u} + \nabla \otimes \mathbf{u}^T\right) + \mathbf{g} \tag{2}$$

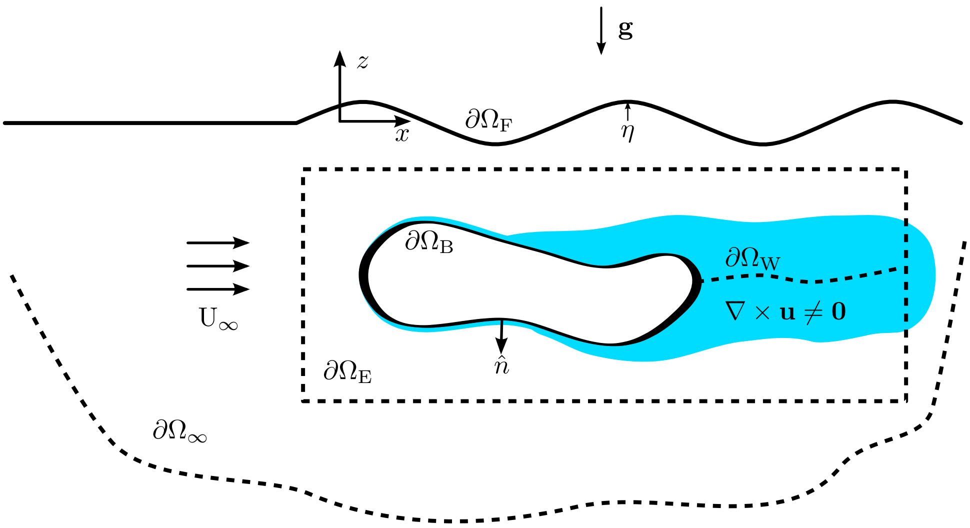

Figure 1: A description of the velocity decomposition approach.

Velocity Decomposition In our approach, we seek a velocity potential which satisfies the real fluid problem in the regions where the flow has no rotation. To arrive at this, we first express the total velocity, $\mathbf{u}$, as the sum of the gradient of the viscous potential, φ, and the vortical component of velocity, $\mathbf{w}$.

$$\mathbf{u} = \nabla\varphi + \mathbf{w} \tag{3}$$

If a viscous potential is found that delivers a vortical velocity field which vanishes with the vorticity vector, then the total velocity can be described completely by the gradient of the viscous potential outside of the vortical regions.

$$\mathbf{u} = \nabla\varphi \qquad \text{on} \qquad \partial\Omega_{\mathrm{E}} \tag{4}$$

This allows us to solve the Navier-Stokes equations on a reduced computational domain which *only* includes the regions where the fluid has rotation. We can then use the viscous potential as a Dirichlet boundary condition on the boundary of this reduced domain. The velocity decomposition approach is described in Figure 1, where $\partial\Omega_{\mathrm{E}}$ is the boundary of the reduced domain.

Navier-Stokes Sub-Problem The Navier-Stokes equations are solved using OpenFOAM, an open-source, finite-volume, CFD package. The steady fluid equations are solved using the SIMPLE algorithm. The Spalart-Allmaras turbulence model with wall functions is used when needed. It is also important to mention that the potential flow panel method uses the same discretization of the body as the viscous flow solver.

Viscous Potential Sub-Problem The viscous potential is solved using a conventional boundary-element method. Flat panels are distributed on the body and wake surfaces. The body is represented using linearly varying vortex panels and constant strength source panels. The constant strength source panels satisfy the body-boundary condition for the viscous potential (5) from Edmund (2012).

$$\frac{\partial\varphi}{\partial n} = -\int_0^\delta \frac{\partial w_t}{\partial t}\,\mathrm{dn} \qquad \text{on} \qquad \partial\Omega_{\mathrm{B}} \tag{5}$$

Where $\hat{\mathbf{n}}$ and $\hat{\mathbf{t}}$ are the unit normal and tangential vectors in a body-fixed coordinate system, w_t is the tangential component of the vortical velocity, and δ is the distance from the body in the normal direction at which the vorticity is negligible.

The linearly varying vortex panels satisfy the inviscid body-boundary condition and are used to solve the lifting problem. In a conventional lifting potential flow approach, the Kutta condition is used to ensure smooth flow at the trailing edge and effectively sets the value of the body-bound circulation. The lift force on the body in a potential flow can be related to the circulation through the Kutta-Joukowski theorem. However, in a viscous flow where separation may be be present, the lift on the body is less than the lift calculated using an inviscid approach. Therefore, the conventional Kutta condition is exchanged in favor of a condition on the total circulation on the body, (6). Where $\mathcal{L}^{NS}$ is the lift on the body from the Navier-Stokes solver.

$$\sum_{m=0}^{N_{\text{body}}} \gamma_m \cdot l_{\text{panel},\ m} = \Gamma^{NS} = \frac{\mathcal{L}^{NS}}{\rho \mathrm{U}_\infty} \tag{6}$$

The body-boundary condition for the viscous potential involves an integral in the normal direction. However, the wake surface has two normal vectors and the result of the integral in (5) will be different for each direction in an asymmetrical flow. This presents a problem because we have two different values of the boundary condition that must be satisfied at the same collocation point. To resolve this problem, we use coincidentally located source and dipole distributions. The source distribution satisfies the average value of the integrals while the dipole distribution satisfies the difference. In other words, the source distribution represents the average thickness of the wake and the dipole distribution incorporates the asymmetrical nature of the wake in a lifting flow.

Finally, the free surface is discretized using desingularized point sources above the mean free surface. The viscous potential satisfies the steady, linear combined free-surface condition in (7).

$$\phi_{xx} + \frac{g}{{\mathrm{U}_\infty}^2}\phi_z = 0 \qquad \text{on} \qquad z = 0 \tag{7}$$

Results

Velocity decomposition results are computed to compare with experimental and numerical results presented in Salvesen (1966) for the free-surface flow over a thick ($t/c = 0.34$) foil at $\alpha = 0°$. The computational domain of the Navier-Stokes sub-problem extends from $x_{\text{in}} = 1c$ to $x_{\text{out}} = 40c$ with the lateral boundary located at $x_{\text{extent}} = 1c$ from the body. Following the experimental setup, the leading edge of the foil is located at $(x, z) = (-0.05c, -1.25c)$. Three different values of $\mathrm{Fn_c} = \mathrm{U}_\infty/\sqrt{gc}$ are tested.

The agreement to experiments is acceptable for all three cases considering that there is likely error present in the original experimental measurement as well as in the process of reproducing the data from the original paper. It is interesting that in some cases the amplitude of the free-surface waves calculated from the viscous potential is larger than the inviscid results, while in other cases the opposite occurs. It is possible that this is due to the lift on the body. In the cases where the free-surface amplitude is smaller for the viscous potential, the lift is positive. The opposite is true for the cases in which the amplitude of the free-surface waves calculated from the viscous potential is larger. Also, the higher-order inviscid free-surface elevation agrees better with the experiments than the linear viscous potential result for most cases. This suggests that the implementation of a higher-order free-surface boundary condition would improve the results.

Conclusion

The velocity decomposition approach has been applied to the solve for the free-surface flow over a fully-submerged thick foil. The comparison to previous experimental and numerical results in Salvesen (1966) is acceptable but could be improved, possibly with a higher-order free-surface boundary condition.

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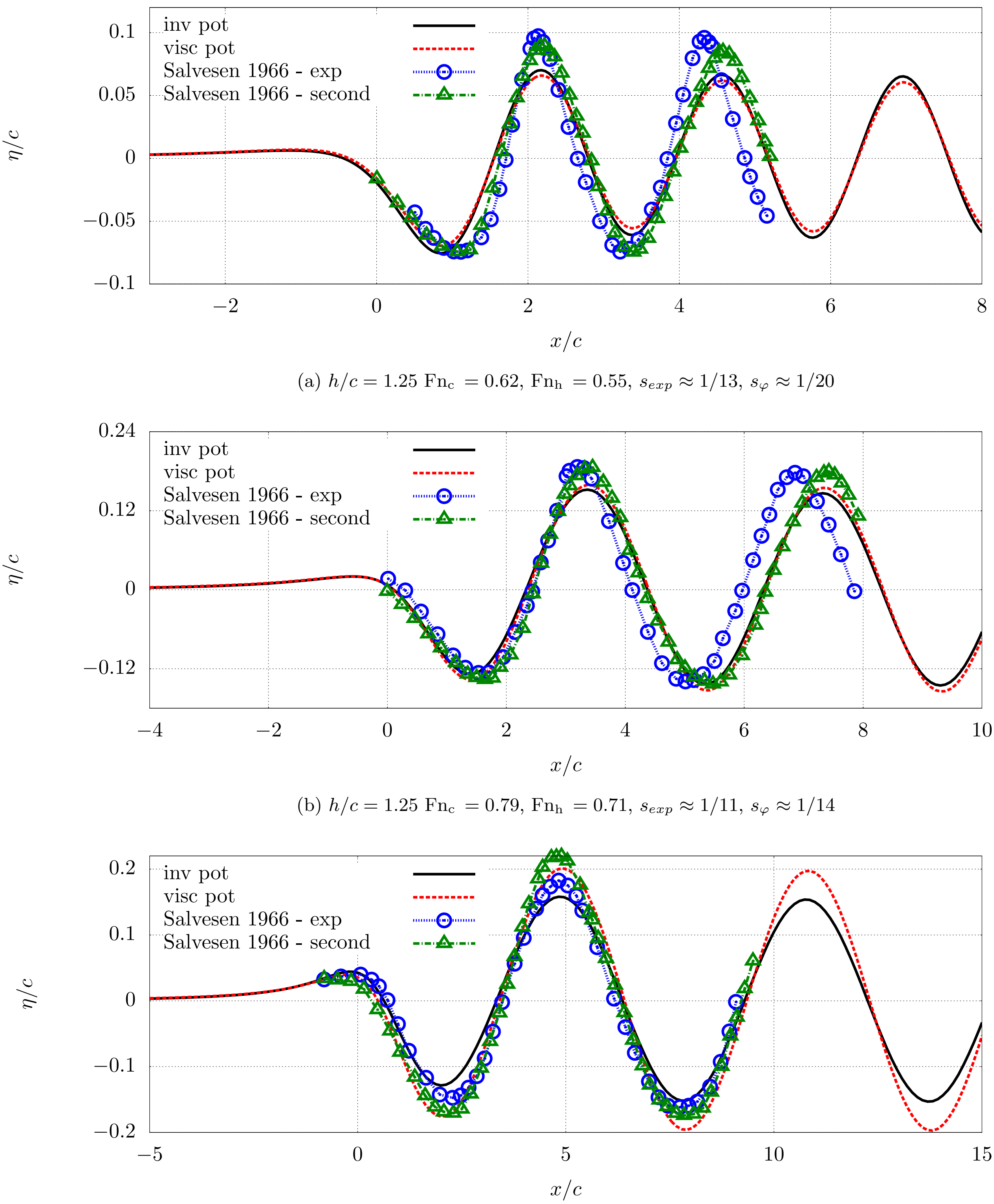

(a) $h/c = 1.25$ $\mathrm{Fn_c}$ $= 0.62$, $\mathrm{Fn_h}$ $= 0.55$, $s_{exp} \approx 1/13$, $s_\varphi \approx 1/20$

(b) $h/c = 1.25$ $\mathrm{Fn_c}$ $= 0.79$, $\mathrm{Fn_h}$ $= 0.71$, $s_{exp} \approx 1/11$, $s_\varphi \approx 1/14$

(c) $h/c = 1.25$ $\mathrm{Fn_c}$ $= 0.97$, $\mathrm{Fn_h}$ $= 0.87$, $s_{exp} \approx 1/16$, $s_\varphi \approx 1/19$

Figure 2: Free-surface profiles for a range of $\mathrm{Fn_c}$ at $h/c = 1.25$ compared to the numerical and experimental results from Salvesen (1966), where s is the approximate wave steepness.