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Improvement of Rankine Panel Method by Theoretical Consideration of Panel Forces on Ship Hull

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Highlights

- New considerations are implemented for the suitable model as the higher order evaluation in the integration equation of Rankine Panel Method, which is expected to apply for the highly accurate practical model.
- Higher Order Sources and Normal Dipoles Method is the best model for advantages in the numerical procedure at singular panels and is little influeces by the second order derivations on the free water surface condition.

1. Introduction

Nowadays, our daily life cannot be sustained without global ship transportation, which carries more than 95 percents of these cargoes, thus a cost effective and safety transportation are strictly required. Weather routing is the one of the most important themes. When we consider the international voyage across the ocean, the possibility of encountering stormy weathers cannot be ruled out. The evaluation of ship performance in heavy seas must be more accurate than ever. Although there are many studies on seekeeping theories, they have not practically used. The complexity of theories may influences to the situation. RIOS(The Research Initiative on Oceangoing Ships)[1] has constructed on the theory of EUT(Enhanced Unified Theory)[2] recently, it seems the first step as the high accuracy practical model. Kashiwagi et al. also tried a theoretical improvement of the EUT for high speed vessels. However, effective improvement cannot be achieved yet[2]. Three dimensional methods are theoretically prior than two dimensional methods, however, source points must be evaluated as panels, not lines. It means that numerical influences are more sensitive where the shape is complex, such as in bow and aft parts of vessels. Constant Panel Method (CPM) is commonly used to solve the integral equation now, matrix elements is represented as uniform values on a panel. There are also some studies on the higher order evaluation of panel integration in frequency analysis without forward speed, or time domain analysis with forward speed. However, there are very few on frequency analysis with forward speed. Rankine Panel Method (RPM) is expected to apply as the new practical method, however, current version is modeled by CPM. There are some possibilities to improve the accuracy by introducing higher order evaluation models. Here, theoretical background is summarized and considered the suitable theoretical method to RPM.

2. Characteristics of Contant Panel Method in Rankine Panel Method

RPM is the three dimensional theory, its kernel function as the fundamental solution term of $1/r$. r is the distance between the field point $P(x, y, z)$ and the panel point $Q(\xi, \eta, \zeta)$ as $r = \sqrt{(\xi - x)^2 + (\eta - y)^2 + (\zeta - z)^2}$. It means that the kernel function does not contain free-surface Green function term, which is related on free water surface and far field radiation conditions. Formulations in free water surface become necessary, in addition to that in body surface. The Green function method, which is strictly theoretical model, is also rather difficult to consider the double body flow into the free-surface term. RPM can consider it by applying directly free water surface condition into the integration equation, thus it is expected to be the practical model. It is also necessary to consider the integration terms in far field areas in a strictly theoretical meanings, however, it is impossible to define them in numerical modeling. The panel shift technique, which is the one column shift of panels on free water surface, is often used to satisfy this condition. Numerical techniques are sometimes required for the complicity of radiation and diffraction waves, especially vessels with forward speed. Figure 1 shows the comparison of diffraction waves (cos component) around the modified wigley model ship, between model test and numerical simulation. Simulated waves underestimate observed values, especially near the ship bow. This point strongly relates to the underestimation of the added resistance. It is thought to be insufficient for the evaluation of panel integrations here. CPM cannot consider the continuity of values for neighboring panels, especially in remarkable shape variation parts. The integration equation in case of a direct method is expressed as follows.

$$2\pi\phi(P) - \iint_{S_H+S_F} \phi(Q) \frac{\partial}{\partial n_Q} \left(\frac{1}{r}\right) dS(Q) = - \iint_{S_H+S_F} \frac{\partial \phi(Q)}{\partial n_Q} \left(\frac{1}{r}\right) dS(Q) \qquad (1)$$

where, ϕ shows the unknown velocity potential, n_Q is the normal vector on the panel Q, $dS(Q)$ shows the area of the panel Q, S_H and S_F show body surface and free water surface, respectively. In RPM with indirect method, the velocity potential is expressed by source and kernel function, as follows.

$$\phi(P) = \iint_{S_H+S_F} \sigma(Q)G(P,Q)dS \tag{2}$$

where, $\sigma(Q)$ shows source function as unknown parameter, and $G(P,Q)$ shows the kernel function. RPM considers the double body flow on S_H, the kernel function considers mirror panels against the water surface. Eq.(2) can be transformed as follows.

$$\phi_j(P) = -\iint_{S_H} \sigma_j(Q)\left\{G_1(P,Q) + G_2(P,Q)\right\} dS - \iint_{S_F} \sigma_j(Q)G_1(P,Q)dS \tag{3}$$

where, $G_1(P,Q)$ and $G_2(P,Q)$ are kernel functions on ship hull and on mirror image of them as follows.

$$G_1(P,Q) = \frac{1}{4\pi r}, G_2(P,Q) = \frac{1}{4\pi r'} \tag{4}$$

$$r' = \sqrt{(\xi - x)^2 + (\eta - y)^2 + (\zeta + z)^2} \tag{5}$$

Here, the free surface condition considering the double body flow, by Sclavounos and Nakos[3], is used as follows.

$$\begin{aligned} -K_e\phi_j(P) + 2i\tau\overline{\nabla}\Phi_D(P)\cdot\overline{\nabla}\phi_j(P) + \frac{1}{K_0}\overline{\nabla}\Phi_D(P)\cdot\overline{\nabla}\left(\overline{\nabla}\Phi_D(P)\cdot\overline{\nabla}\phi_j(P)\right) \\ +\frac{1}{2K_0}\overline{\nabla}\left(\overline{\nabla}\Phi_D(P)\cdot\overline{\nabla}\Phi_D(P)\right)\cdot\overline{\nabla}\phi_j(P) + \frac{\partial\phi_j(P)}{\partial z} = 0 \quad \text{on} \quad z = 0 \end{aligned} \tag{6}$$

where, K_e shows the wave number expressed by the encounter frequency, ω_e and $K_e = \omega_e/g^2$, K_0 shows incident wave number , $\Phi_D(P)$ shows steady velocity potential in double body flow at P, $\overline{\nabla}$ means the partial differentiation with x and y. The body surface condition is also defined for radiation problem (suffix $j = 1-6$) and diffraction problem (suffix $j = 7$).

$$\frac{\partial\phi_j(P)}{\partial n} = n_j + \frac{U}{i\omega_e}m_j \quad (j = 1 \sim 6), \quad \frac{\partial\phi_7(P)}{\partial n} = -\frac{\partial\phi_0(P)}{\partial n} \quad \text{on} \quad S_H \tag{7}$$

where, m_j shows influence term in steady flow for j mode motion, n_j shows normal vector component for j mode motion, ϕ_0 shows the velocity potential of incident wave ($\phi_0 = ie^{k_0 z - ik_0(x\cos\chi + y\sin\chi)}$). Integral equations can be obtained, if Eq.(2) is substituted to Eqs.(6)-(7). It is obvious that there are some terms of the second order differential. If the direct method as Eq.(1) is used here, coefficients become zero or constant values theoretically because the velocity potential is expressed as the first order or the second order functions. Based on this point, advantages and disadvantages of following methods will be considered here.

3. Higher Order Panel Evaluation

When the integration equation (1) is discretized into N panels, it is expressed as follows.

$$2\pi\phi(P_i) - \sum_{j=1}^{N}\phi(Q_j)\mathcal{D}_{ij} = -\sum_{j=1}^{N}\left(\frac{\partial\phi}{\partial n}\right)_j \mathcal{S}_{ij} \qquad (i = 1 - N) \tag{8}$$

$\mathcal{D}_{ij}$ and $\mathcal{S}_{ij}$ are expressed in CPM as follows.

$$\mathcal{D}_{ij} = \iint_{S_j} \frac{\partial}{\partial n}\left(\frac{1}{r}\right) dS(Q), \quad \mathcal{S}_{ij} = \iint_{S_j} \frac{1}{r} dS(Q) \tag{9}$$

where, S_j shows the discretized j-th panel. There are some methods to evaluate as the higher order model, the possibility of following three models is considered for the application to RPM.

3.1 Quadratic Boundary Element Method

If the body shape of floating structures are difficult to express as the sum of flat panels, nonlinear mode functions are often used. Velocity potentials are also expressed as the multiply of nonlinear mode function and potential in panel node as follows.

$$\phi_j(x,y,z) = \sum_{k=1}^{m} N_k(\xi,\eta)\phi_{jk}(x_k,y_k,z_k) \tag{10}$$

where, (x,y,z) show coordinate of arbitrary point in the panel, (x_k,y_k,z_k) show coordinate of panel node, (ξ,η) show coordinate normalized panel node at the water surface, m is the number of node, and $N_k(\xi,\eta)$ shows mode functions of the second order. Mode functions are expressed as the second order polynomials for 7 and 9 nodes of triangular and quadrilateral panels. Figure 2 shows the relation of node number and defined panels. As shown here, an element in the integration equation consists of 4 neighboring panels. It shows that the redefinition of panels and nodes becomes necessary, so N in Eq.(8) means the number of redefined panels. If Eq.(10) is substituted into Eq.(8), the integration equation as the second order expression can be modeled.

3.2 Cubic B-Spline Galerkin Method

Unknown velocity potential is modeled as the combination of the cubic B-Spline functions[4], shown as follows.

$$\phi(x,y,z) = \sum_{j=1}^{N+3} \alpha_j B_j(P,Q) \tag{11}$$

where, $B_j(P,Q)$ is the third order B-Spline function with parameters of P and Q. They are obtained by Boor-Cox's recurrence formula using coordinates include 4 neighboring panel nodes. If B-Spline functions, $B_k(P,Q)$ are multiplied to the integration equations and integrate them for the body surface, simultaneous equations for unknown coefficients, α_j, can be derived as follows.

$$\sum_{j=1}^{N+3} \alpha_j \left\{ 2\pi \iint_{S_H+S_F} B_k(P,Q)B_j(P,Q)dS - \iint_{S_H+S_F} B_k(P,Q) \iint_{S_H+S_F} B_j(P,Q)\mathcal{D}_{ij}dSdS \right\} = -\mathcal{R}_j \tag{12}$$

where, the right hand term, $\mathcal{R}_j$ shows as follows.

$$\mathcal{R}_j = \iint_{S_H+S_F} B_k(P,Q) \sum_{j=1}^{N+3} \left(\frac{\partial\phi}{\partial n}\right)_j \mathcal{S}_{ij} dS \tag{13}$$

3.3 Arbitrary Higher Order Sources and Normal Dipoles Method

Newman derived the evaluation of the integral equation with linear and nonlinear kernel functions[5]. They are defined as the zero order elements, Ψ_{00} and Φ_{00}, corresponding to $\mathcal{D}$ and $\mathcal{S}$ in Eq.(8).

$$\Psi_{00} = \iint_{S_H+S_F} \frac{1}{r} d\xi d\eta, \quad \Phi_{00} = \iint_{S_H+S_F} \left[\frac{\partial}{\partial\zeta}\left(\frac{1}{r}\right)\right]_{z=0} = z\iint_{S_H+S_F} \frac{1}{r^3} d\xi d\eta \tag{14}$$

These integrations are theoretically modeled for numerical analysis, without special procedures in singular panels. Details are not described here, see the reference[5]. Then, linear and bilinear components are derived as follows.

$$\begin{pmatrix} \Psi_{10} \\ \Psi_{01} \end{pmatrix} = \iint_{S_H+S_F} \begin{pmatrix} \xi \\ \eta \end{pmatrix} \frac{1}{r} d\xi d\eta = \begin{pmatrix} x \\ y \end{pmatrix} \Psi_{00} \mp \sum_{n=1}^{4} \begin{pmatrix} \cos\theta_n \\ \sin\theta_n \end{pmatrix} \int_{S_n} r d\ell \tag{15}$$

$$\begin{pmatrix} \Phi_{10} \\ \Phi_{01} \end{pmatrix} = z\iint_{S_H+S_F} \begin{pmatrix} \xi \\ \eta \end{pmatrix} \frac{1}{r^3} d\xi d\eta = \begin{pmatrix} x \\ y \end{pmatrix} \Phi_{00} \pm z\sum_{n=1}^{4} \begin{pmatrix} \sin\theta_n \\ \cos\theta_n \end{pmatrix} \int_{S_n} \frac{1}{r} d\ell \tag{16}$$

$$\Psi_{11} = \iint_{S_H+S_F} \frac{\xi\eta}{r} d\xi d\eta = \sum_{n=1}^{4} \cos\theta_n \int_0^{S_n} r\left(\xi_n - x + s\cos\theta_n\right) ds + x\Psi_{01} + y\Psi_{10} - xy\Psi_{00} \tag{17}$$

$$\Phi_{11} = z\iint_{S_H+S_F} \frac{\xi\eta}{r^3} d\xi d\eta = z\sum_{n=1}^{4} \cos\theta_n \int_0^{S_n} \frac{\xi_n - x + s\cos\theta_n}{r} ds + x\Phi_{01} + y\Phi_{10} - xy\Phi_{00} \tag{18}$$

where, θ_n shows the n-th angle between the horizontal axis and the panel node, s shows distance between panel nodes, (x,y,z) is the field point, and (ξ,η) is the panel node. Values in higher orders can be obtained by following recurrence formulas.

$$\Phi_{m+2,n} = (m+1)\Psi_{m,n} - \iint_{S_H+S_F} \frac{\partial}{\partial\xi}\left[\frac{(\xi-x)^{m+1}(\eta-y)^n}{r}\right] d\xi d\eta \tag{19}$$

$$\Phi_{m,n+2} = (n+1)\Psi_{m,n} - \iint_{S_H+S_F} \frac{\partial}{\partial \eta}\left[\frac{(\xi-x)^m(\eta-y)^{n+1}}{r}\right] d\xi d\eta \tag{20}$$

$$(m+n+1)\Psi_{mn} + z^2\Phi_{mn} = \iint_{S_H+S_F} \left\{\frac{\partial}{\partial \xi}\left[\frac{(\xi-x)^{m+1}(\eta-y)^{n}}{r}\right] + \frac{\partial}{\partial \eta}\left[\frac{(\xi-x)^m(\eta-y)^{n+1}}{r}\right]\right\} d\xi d\eta \tag{21}$$

The evaluation of arbitrary order becomes available, by the iteration of Eqs.(19)-(21) based on Eqs. (15)-(18).

4. Future Modeling as Practical Accurate Model

In the Quadratic Boundary Element Method, coefficients may become constant values in the free water surface part for the second order function in the direct method. The numerical procedure is necessary at the singular panel, although complicated body shape in ship bow can be accurately considered. In the B-Spline Galerkin Method, coefficients become linear functions theoretically in the free water surface part because of the third order function in the direct method. If Cubic B-Spline functions can be modeled as orthogonal functions, terms in the integral equation is expected to be simpler and reduce simulation time. The computation time will be enormous, if the orthogonal relation cannot be used. In the Arbitrary Higher Order Sources and Normal Dipoles Method, the numerical procedure is not necessary in singular panels because of mathematical derivation technique. It is also possible to define arbitrary order formulations can be available, if the recursion formulas are repeatedly used, although the body shape must be reproduced as the sum of flat panels. There are advantages and disadvantages in each method, however, the Arbitrary Higher Order Sources and Normal Dipoles Method is the most suitable because of strong theoretical background and the numerical procedure in singular points. Authors are implementing this numerical modeling, and consider the improvement for high accuracy practical model.

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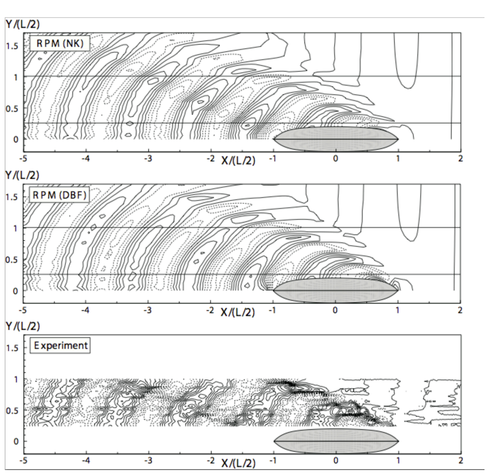

Figure 1: Comparison of diffraction waves

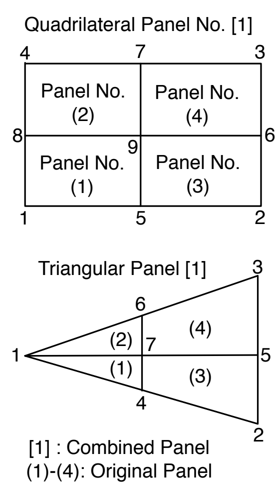

Figure 2: Panel redefinition from 4 panels